Variational Methods in Celestial Mechanics

organized by
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Workshop Summary

The conference succeeded admirably in its goal of fostering and deepening interactions between researchers in calculus of variations proper (such as Terracini and Coti-Zelati) and celestial mechanicians (such as Moeckel and Meyer).

A number of people from outside these two domains, but with deep connections to them, were exposed and inspired by the results obtained and the problems presented. They also gave their own perspective, significantly broadening the scope of the meeting and the caliber of problems considered. One example is the pair Pujals and Florit, from IMPA, who came because they wanted to learn the subject and try to develop it in their home institution. During the conference, Pujals played with a proposal to use Marchal's theorem on the fixed endpoint problem as a basis for building some type of closing lemma in configuration space, Hsiang, essentially a geometer, forcefully espoused use of the Jacobi-Maupertuis metric, and in doing so inspired Terracini, Venturelli, Ferrario, and Montgomery to continue trying to get these Jacobi methods to work. For example, if the eight could be characterized as a Jacobi minimizer between its Euler configurations, then we would have a proof that it minimizes within a smaller symmetry class. Kapela and Zglyczinski came from Poland and they presented the basic ideas of their interval arithmetic proofs for the existence and local uniqueness of certain orbit types.

Two graduate students Celli, and Barutello attended and spoke. It was Celli's and Barutello's first public talks. The talks were successful. Barutello's work inspired a program of equivariant gradient flow which may lead to better understanding of how the eight orbit can be deformed into Lagrange. The pictures so obtained by Barutello are related to those of Jacques Fejoz for continuing the eight in a rotating frame.

Concrete results: One proof was nearly completed during the conference: the convexity of each lobe of the figure eight orbit. In the week following the conference that proof was completed. Marchal presented a striking theorem, inspired by the occasion of the conference, asserting that any Hill-stable three-body systems (equivalently, for Dziobek constant large enough that the hill region is topologically three disconnected discs) suffers infinitely many syzygies.