# TITLES AND ABSTRACTS

**Samuel Patterson**
"The context of Riemann's paper on the distribution of prime numbers".
Abstract:

This talk will be historical and almost entirely devoted to developments in mathematics which were the technical background for Riemann's paper.
Riemann, as we know from his Habilitationsschrift, his lectures on differential equations and his paper on Nobili rings, was interested in "summation theorems", many of which are now subsumed in general theorems of functional analysis. The seminal text in this direction in Riemann's day was Fourier's "Théorie de Chaleur" which itself builds on the work of Daniel Bernoulli, Leonard Euler and others. In his 1987 lecture "Prehistory of the Riemann zeta function", held at the conference for Atle Selberg's 70th birthday, André Weil concentrates on the analytic continuation of the zeta function. The aspects which we shall discuss here illuminate his "irrational exuberance" of 1859 concerning the zeros of the zeta function. I shall also argue that by 1863 this had worn off and Riemann was no longer convinced that his methods could explain the empirical data.

**Nick Katz**
"RH in characteristic *p*; the importance of family values"
Abstract:

A historical discussion of RH over finite fields, with an explanation of how families enter in Deligne's proof.

**Paul Garrett (joint with E. Bombieri)**
"Pseudo-Laplacians on automorphic forms"
Abstract:

We report on progress relating the spectral theory of certain self-adjoint operators on spaces of automorphic forms to location of zeros of zeta functions. First, as approximately suggested by Colin de Verdière 1983, the discrete spectrum $\lambda_s = s(s-1)$, *if any*, of a certain pseudo-Laplacian $\widetilde{\Delta}$, attached to a complex quadratic extension $k$ of $\mathbb{Q}$, on the space spanned by pseudo-Eisenstein series, occurs *only* for $\zeta_k(s) = 0$. The proof uses spectral expansions of automorophic distributions. Using the discretization of the continuous spectrum from Faddeev 1967 and Lax-Phillips 1976, and expansion in terms of the corresponding exotic eigenfunctions, we prove that Montgomery's pair correlation would restrict the fraction of zeros of $\zeta(s)$ appearing in such a discrete spectrum to at most 94%. A subtler application of exotic eigenfunction expansions also yields a spacing result for all zeros of $\zeta_k(s)$.

**Enrico Bombieri (joint with P.Garrett)**
"Pseudo-Laplacians: A Special Case"
Abstract:
This lecture is the second part of a joint lecture with Paul Garrett, dealing with a variant of Colin de Verdiere's potential approach to an interpretation of the Riemann hypothesis using the hyperbolic laplacian in the Friedrichs extension of a suitable Hilbert space. It deals explicitly and rigorously with the problem, showing at the end why it is unlikely for such methods to give directly information on the zeta function along such lines. A new problem on quadratic mean-values is also proposed at the end.

All work is in collaboration with Paul Garrett.

**Wei Zhang**
"Positivity of *L*-functions and "completion of square"
Abstract:
We will first explain how Riemann hypothesis and its generalization imply the positivity of all derivatives of self-dual (normalized) *L*-functions at the center of functional equations. This raises a natural question: can we interpret these positive values as "squares"? We present some examples of such interpretations in terms of arithmetic geometric objects. The first example is the Gross-Zagier type formula. Thanks to the Hodge index theorem for arithmetic surfaces, this implies the positivity of the first derivatives of *L*-functions attached to elliptic curves over $\mathbb{Q}$ (or more generally, to certain automorphic representations over a totally real field). In another example, the joint work of the author with Zhiwei Yun expresses higher central derivatives of *L*-functions over function fields in terms of intersection numbers on the moduli space of Drinfeld Shtukas.

**Terry Tao**
"Bounding the de Bruijn-Newman constant"
Abstract:
The Riemann hypothesis is equivalent to the assertion that the entire function $H_0(z) = 1/8\ \xi(1+iz/2)$ has all zeroes on the real line. De Bruijn and Newman studied the deformations $H_t$ of this entire function under the backwards heat equation $\partial_t H_t(z) = -\partial_{zz} H_t(z)$, and showed that there is a real number $\Lambda$, known as the de Bruijn-Newman constant, such that all the zeroes of $H_t$ are real if and only if $t \geq \Lambda$. Thus the Riemann hypothesis is equivalent to the assertion $\Lambda \leq 0$. With Brad Rodgers, we have recently established the complementary bound $\Lambda \geq 0$ (improving upon the previous lower bound of $-1.1 \times 10^{-11}$, and in an ongoing ``Polymath'' collaboration we are improving the previous upper bound of 1/2. The former results rely primarily on an analysis of the dynamics of zeroes under heat flow, and the latter on efficient numerical verifications of zero-free regions for the $H_t$; we will present both of these arguments in this talk.

**Andrew Booker**
"L-functions"
Abstract:

In 1989, Selberg introduced his eponymous class of *L*-functions, giving rise to a new subfield of analytic number theory in the intervening quarter century. Despite its popularity, a few things have always bugged me about the definition of the Selberg class. I will discuss these nitpicks and describe an attempt at reformulating the Selberg class in terms of distributional identities akin to Weil's "explicit formula". Along the way I will wax philosophical about RH and the converse theorem for automorphic forms.

**Will Sawin**
"More on zeroes of *L*-functions over function fields"
<u>Abstract:</u>

The Riemann hypothesis over function fields is a theorem of Weil, with a more general form due to Deligne. This raises the hope of answering more precise questions on the zeroes of *L*-functions, such as the distribution of the zeroes in a suitable family of *L*-functions. Powerful results on this have been proved, starting with the book of Katz and Sarnak, but the theory is not yet complete, and it is an area of active research. I will survey the history, explain recent developments, and discuss what is in reach for the future.

**Alexandra Florea**
"Moments of *L*-functions"
<u>Abstract:</u>

I will discuss the problem of computing moments in families of *L*-functions. I will describe the existing conjectures for moments and present previous results, both in the number field and function field setting.

**Peter Sarnak**
"Commentary and comparisons of some approaches to GRH."

**Kannan Soundararajan**
"The value distribution of zeta and *L*-functions."
<u>Abstract:</u>

I will discuss some topics related to the value distribution of zeta and *L*-functions, beginning with Selberg's theorem on the log normality of the zeta function on the critical line. Analogues of Selberg's theorem for central values of *L*-functions in families are still conjectural. I will describe some recent work in this direction, and explain how these inform our understanding of moments and extreme values.

**Fernando Rodriguez Villegas**

"Hodge numbers of Hypergeometric Motives"

Abstract:

Hypergeometric Motives yield a large class of *L*-functions that are computable and cover a significant range of possible parameters. In this talk I will concentrate on the distribution of their Hodge numbers as an approach to the larger question of how Hodge numbers of motivic *L*-functions are distributed.

**Alain Connes**

"The Riemann-Roch strategy" (joint work with C. Consani)

Abstract:

I will explain in my talk the slow progression in the understanding of a geometry allowing one to get the explicit formulas as a trace formula, the zeta function as a Hasse-Weil counting and to start transposing the proof of Weil using a Riemann-Roch formula following Mattuck, Tate and Grothendieck. The work extends over twenty years and is joint work with Consani in the last ten years. During this last period we found the topos theoretic interpretation of the adele class space as the space of points of the scaling site, ie the semidirect product of the half line by multiplication by positive integers.

This site comes endowed with the structure sheaf of piecewise affine convex functions with integral slopes: a geometric structure of tropical type. We use mathematics in characteristic one to obtain Riemann-Roch formulas in this context, and generalizations of the Jensen formula to almost periodic functions in order to relate the existence part in Riemann-Roch to known results in complex geometry. My talk will be a survey of this attempt at RH.

**Brian Conrey**

"*L*-functions and Random Matrix Theory"

Abstract:

We give an overview of some of the interactions between RMT and the study of statistics of *L*-functions.

**Christopher Skinner**

"Zeros of *L*-functions and ranks of elliptic curves"

<u>Abstract:</u>

Along with the zeros of the Riemann zeta function and the elusive Siegel zeros, the zeros at $s$=1 of the L-functions $L(E,s)$ of elliptic curves E have fascinated number theorists for many years. This talk will describe some of what is known about the arithmetic significance of the values $L(E,1)$ and various generalizations.

**Maksym Radziwill**

"Typical behavior of *L*-functions"

<u>Abstract:</u>

I will discuss various results and conjectures on the ``typical behavior'' of *L*-functions, both in discrete and continuous families, and explain their arithmetic significance.

**Keith Ball**

"Rational approximations to ζ "