# *Handbook of Linear Algebra, 1st Edition*
# Errata List

September 5, 2015

Changes are shown in red. Minor grammatical/spelling corrections do not appear on this list unless they could cause confusion.

**Corrections and clarifications involving chapter interaction:**

- The definition of *sparse* varies with the chapter. Specifically, in Chapter 43 sparse means few nonzero entries, whereas in Chapter 40 it means few nonzero entries and in addition that it behaves well under gaussian elimination. See also correction below to the definition of sparse in the Glossary.
- A number of references to other chapters are wrong. These are listed individually below.

**Corrections and clarifications within a single chapter (in order):**

Preliminaries p. xxvi, Fact 1 (under Complex Numbers) should be:
$|c| = \sqrt{c\bar{c}}$.

Preliminaries p. xxvii, Equivalence Relation third part of definition should be:
3. (Transitive) For all $a, b, c \in S$, $a \equiv b$ and $b \equiv c$ imply $a \equiv c$.

Preliminaries p. xxix, Definition of *little oh* should be:
$f$ is $o(g)$ (**little-oh** of $g$) if $\lim_{x\to\infty} \left|\frac{f(x)}{g(x)}\right| = 0$.

Ch. 1 p. 1-3, line -2, should be:
and a **column vector**, respectively, and they belong to $F^{1\times n}$ and $F^{n\times 1}$, respectively. The elements of $F^n$ are

Ch. 1 p. 1-9, line -10, should be:
variables, such as $\begin{array}{l} a_{11}x_1 + \cdots + a_{1p}x_p = b_1 \\ a_{21}x_1 + \cdots + a_{2p}x_p = b_2 \\ \cdots \\ a_{m1}x_1 + \cdots + a_{mp}x_p = b_m \end{array}$. A **solution** of the system is a $p$-tuple $(c_1, \ldots, c_p)$ such that

Ch. 1 p. 1-9, line -5, should be:
For the system $\begin{array}{l} a_{11}x_1 + \cdots + a_{1p}x_p = b_1 \\ a_{21}x_1 + \cdots + a_{2p}x_p = b_2 \\ \cdots \\ a_{m1}x_1 + \cdots + a_{mp}x_p = b_m \end{array}$, the $m \times p$ matrix $A = \begin{bmatrix} a_{11} & \cdots & a_{1p} \\ \vdots & \ldots & \vdots \\ a_{m1} & \cdots & a_{mp} \end{bmatrix}$ is the **coefficient**

Ch. 2 p. 2-3, Application 1, last sentence should be:
In general, the solution space for a linear homogeneous differential equation is a vector space, meaning that any linear combination of solutions is again a solution.

Ch. 2 p. 2-11, Examples, symbols for the change of basis matrix are wrong in several examples:
In Example 2 (three times) the symbol for the the change of basis matrix should be ${}_{\mathcal{E}_2}[I]_{\mathcal{B}}$.
In Example 3 (three times) the symbol for the the change of basis matrix should be ${}_{\mathcal{B}}[I]_{\mathcal{B}'}$
In Example 4 the symbols for the the change of basis matrices should be ${}_{\mathcal{B}_2}[I]_{\mathcal{B}_1}$, ${}_{\mathcal{E}_2}[I]_{\mathcal{B}_2}$, ${}_{\mathcal{E}_2}[I]_{\mathcal{B}_1}$.

Ch. 4 p. 4-3, Application 2: The subscripts are reversed in the formula for the determinant of the Vandermonde matrix; it should be:
$\prod_{1\leq i<j\leq n}(x_j - x_i)$.

Ch. 4 p. 4-3, the notation for the $k$th compound matrix is inconsistent with that used subsequently. The definition should be:
The $k$**th compound matrix** of $A \in F^{m\times n}$ is the $\binom{m}{k} \times \binom{n}{k}$ matrix $A^{(k)}$ whose entries are the $k \times k$ minors of $A$, usually in lexicographical order.

Ch. 4 p. 4-4, Fact 5, should be:

5. [A56] $A^{(k)}\left(\text{adj}^{(k)} A\right) = \left(\text{adj}^{(k)} A\right) A^{(k)} = (\det A)\, I_{\binom{n}{k}}$

Ch. 4 p. 4-8, the definition of the characteristic polynomial of a linear operator is wrong. The definition should be:

For a linear operator, $T$, on a finite dimensional vector space, $V$, with a basis, $\mathcal{B}$, the **characteristic polynomial** of $T$ is given by $p_T(x) = p_{[T]_{\mathcal{B}}}(x)$.

Ch. 5 p. 5-10, last sentence gives wrong internal reference. It should be:
(See Chapter 17 for more information on singular value decomposition.)

Ch. 6 p. 6-7, Fact 4 should be:
Provided the real-Jordan blocks $J^{\mathbb{R}}_{2k}(\alpha + \beta i)$ are chosen with $\beta > 0$, $A, B$ are similar if and only if their real-Jordan canonical forms have the same multiset of Jordan and real-Jordan blocks.

Ch. 6 p. 6-9, second sentence of Fact 8 should be:
The primary decomposition is unique up to the order of the monic irreducible polynomials, i.e., the multiset of primary factors of $q(x)$ is unique.

Ch. 6 p. 6-14, Example 3, first paragraph should be:
We can use Algorithm 2 to find a matrix $S$ such that $\text{RCF}_{IF}(A) = S^{-1}AS$ for the matrix $A$ in Example 1 of Section 6.5.

Ch. 9 p, 9-3, Fact 3(c) is wrong. Fact 3 should be:

3. (Characterizing Aperiodicity) [HJ85, §8.5] Let $P$ be an irreducible nonnegative $n \times n$ matrix. The following are equivalent:
   (a) $P$ is aperiodic.
   (b) $P^m > 0$ for some $m$.
   (c) $P^m > 0$ for all $m \geq n^2 - 2n + 2$.
   (d) $P^{n^2-2n+2} > 0$.

Ch. 9 p. 9-10, Fact 6(c), should be:

c) $\rho < \mu$ if and only if $P\mathbf{u} < \mu\mathbf{u}$ for some vector $\mathbf{u} \geq 0$ .

Ch. 10 p. 10-2 last line should be:
(of Example 5) and $B$ are...

Ch. 14 p. 14-1 last line should be:

$$x^n - S_1(A)x^{n-1} + S_2(A)x^{n-2} + \cdots + (-1)^{n-1}S_{n-1}(A)x + (-1)^n S_n(A).$$

Ch. 14 p 14-4 Fact 15 part (a) should be:

$$\lambda_k(A) + \lambda_n(B) \leq \lambda_k(A+B) \leq \lambda_k(A) + \lambda_1(B).$$

Ch. 14 p 14-5 line 3 should be:

$$-1.495 \leq \operatorname{Re}\lambda \leq 2.8484.$$

Ch. 14 p 14-6 second line of Fact 4 should be:
union of the ovals of Cassini of $A$ by $K(A)$ (see Fact 3), we have that

Ch. 16 p. 16-3, Fact 9(a): Delete "-triangular" The first line should be:
If $A$ has the block form

Ch. 16 p. 16-7, Example 2: The second symbol $a(t)$ is wrong. The whole example should be:

2. Pseudospectra of matrices with the symbols

$$a(t) = \mathrm{i}t^4 + t^2 + 2t + 5t^{-2} + \mathrm{i}t^{-5}$$

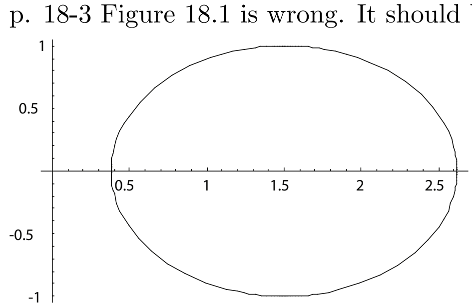
and

$$a(t) = 3\mathrm{i}t^4 + t + 3\mathrm{i}t^{-2}$$

are shown in Figure 16.5

Ch. 17 p. 17-3, Fact 9, delete the extraneous phrase at end of the first line; first line should be:
Let $A \in \mathbb{C}^{m\times n}$.

Ch. 18 p. 18-3 Figure 18.1 is wrong. It should be:




Ch. 19 p. 19-4, line 1, cited Section number is wrong should be:
... (See Section 9.5 for more information and

Ch. 28 p. 28-10, Example 1, last statement is wrong. It should be:
$\eta(G) = 5$ has been shown by computations by Geoff Tims.
The unique strongly regular graph $G$ with parameters (27, 10, 1, 5) (the complement of the Schläfli graph) has $\eta(G) < \chi(\overline{G})$ [Hae79].

Ch. 35 p. 35-3, Fact 8 is wrong. It should be:
Each of the classes $X$ listed at the beginning of the facts except positive $P$-matrices is closed under matrix direct sums.
(Fact 10 is correct [H01].)

Ch. 35 p. 35-9, Fact 6: The first sentence is wrong. It should read:
Let $B$ be a partial positive semidefinite matrix such that every diagonal entry is specified and $\mathcal{G}(B)$ with loops suppressed is a cycle.

Ch. 35 p. 35-14, Fact 2: The second sentence is wrong. It should read:

Then $A = \begin{bmatrix} B_{11} & \mathbf{b}_{12} & \mathbf{b}_{12} b_{22}^{-1} \mathbf{b}_{23}^T \\ \mathbf{b}_{21}^T & b_{22} & \mathbf{b}_{23}^T \\ \mathbf{b}_{32} b_{22}^{-1} \mathbf{b}_{21}^T & \mathbf{b}_{32} & B_{33} \end{bmatrix}$ is ...

Ch. 35 p. 35-14, Fact 5: The second sentence is wrong. It should read:
A strongly connected nonseparable path-clique digraph is homogeneous.

Ch. 35 p. 35-15, Fact 8: The second sentence is wrong. It should read:

Then $A = \begin{bmatrix} B_{11} & \mathbf{b}_{12} & \mathbf{b}_{12} b_{22}^{-1} \mathbf{b}_{23}^T \\ \mathbf{b}_{21}^T & b_{22} & \mathbf{b}_{23}^T \\ 0 & \mathbf{b}_{32} & B_{33} \end{bmatrix}$ is a $P_{0,1}$-completion of $B$.

Ch. 35 p. 35-16, Example 1: There are several errors. Phrases in the numbered line should read:
(lines 1, 2) ... The graph 1a is interpreted as the digraph of B.
(line 4) ... Let $x_{13} = z$ and $x_{31} = -z$. With
(lines 6, 7, 8) ... so setting $z = 1$ completes $B$

to the $P$-matrix $\begin{bmatrix} 1 & 2 & 1 & -1 \\ 1 & 3 & -2 & 2 \\ -1 & 2 & 1 & 1 \\ -1 & -2 & 1 & 2 \end{bmatrix}$. Any partial $P$-matrix specifying 1a, or 1d (the double triangle), or any other symmetric digraph can be completed in a similar manner.

Ch. 35 p. 35-20 Example 3 should read:
The graph 1f has the entry sign symmetric $P$- (entry sign symmetric $P_{0,1}$-, entry sign symmetric $P_0$-, entry weakly sign symmetric $P$-, entry weakly sign symmetric $P_{0,1}$-, entry weakly sign symmetric $P_0$-) completion proper by Fact 5.

Ch. 37 p. 37-9, Fact 3, last sentence could be rephrased as:
If the matrix norm used to compute $\|A^{-1}\|$ is induced by the vector norm used to compute $\|b\|$, then equality is possible.

Ch. 40 p. 40-15, Example 1, 5,5-entry of the triangular factorization of $A$ is wrong. The factorization should be:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -\frac{1}{7} & 1 & 0 & 0 & 0 \\ \frac{1}{7} & -\frac{1}{50} & 1 & 0 & 0 \\ -\frac{1}{7} & \frac{1}{50} & -\frac{1}{51} & 1 & 0 \\ \frac{1}{7} & -\frac{1}{50} & \frac{1}{51} & -\frac{1}{52} & 1 \end{bmatrix} \begin{bmatrix} 7 & 1 & -1 & 1 & -1 \\ 0 & \frac{50}{7} & -\frac{1}{7} & \frac{1}{7} & -\frac{1}{7} \\ 0 & 0 & \frac{357}{50} & -\frac{7}{50} & \frac{7}{50} \\ 0 & 0 & 0 & \frac{364}{51} & -\frac{7}{51} \\ 0 & 0 & 0 & 0 & \frac{371}{52} \end{bmatrix}$$

Ch. 44 p. 44-8, Figure 44.3: The third figure (at the bottom of the page) is duplicate of Figure 44.4 on p. 44-12 and was placed there through a copy-editing error. The first two diagrams are the correct Figure 44.3.

Ch. 46 p. 46-5, last 2 bullets of Fact 10: replace arrow symbol by dagger, to read:
where $\eta_j = \|\delta\tilde{A}^{(j)}(\tilde{A}^{(j)})^\dagger)\|_2 \leq c_j\epsilon\|(B^{(j)})^\dagger\|_2$, and $\sigma_i(\cdot)$ stands for the $i$th largest singular value of a matrix. The accuracy of Algorithm 2 is determined by $\beta \equiv \max_{k\geq 0}\|(B^{(k)})^\dagger\|_2$. It is observed in practice that $\beta$ never exceeds $\|B^\dagger\|_2$ too much.

- If $A$ initially contains column-wise small uncertainty and if $\beta/\|B^\dagger\|_2$ is moderate, then Algorithm 2 computes as accurate approximations of the singular values as warranted by the data.

Ch. 46 p. 46-9, first 2 bullet of Fact 5: last 2 sentences should be:
The upper bound on $\kappa_2(L)$ depends on pivoting $P$ in Step 1 and can be $O(p^{1+(1/4)\log_2 p})$. For the Businger–Golub, pivoting the upper bound is $O(2^p)$, but in practice one can expect an $O(p)$ bound.

Ch. 50 Page 50-3: last line is wrong. It should be:
form. The optimal value (the minimum) is 3. An optimal solution is $x = 1, y = 2$. It is unique

Ch. 54 Page 54-11, last line: replace $A$ with $\alpha$, to read:
where the set $\alpha$ is finite, and $\beta = \mathbb{N} - \alpha$.

Ch. 54 Page 54-13, Algorithm 1, item (c) is wrong. It should be:
(c) for $i = m+1, \ldots, k$ set $a_{i,i} = -\sum_{l=m+1, l\neq i}^{k} a_{i,l}$.

Ch. 59 p. 59-5, Fact 4, first sentence is wrong. It should be:
One can find a basis of solutions $\mathbf{z}_k$ to $A\mathbf{z}_k = \lambda_k B\mathbf{z}_k$, with $\lambda_k$ real.

Ch. 61 p. 61-5, Fact 7, displayed formula is wrong. It should be:

$$(q^{r-1} - 1)/(q - 1) < n \leq (q^r - 1)/(q - 1).$$

Ch. 71 p. 71-1: The version of MATLAB discussed in Chapter 71 is MATLAB 7.0 (also referred to as release 12). Since then they have come out with 3 or 4 new releases, but there do not seem to be any changes in the way matrix computations are done.

Ch. 71 p. 71-4: The 3rd line of Example 1 is missing a period in the second $A * B$. The entire example should read:

1. If

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 5 & 1 \\ 2 & 3 \end{bmatrix}$$

the commands `A*B` and `A.*B` will generate the matrices

$$\begin{bmatrix} 9 & 7 \\ 23 & 15 \end{bmatrix}, \quad \begin{bmatrix} 5 & 2 \\ 6 & 12 \end{bmatrix}.$$

Ch. G p. G-34, **sparse** should be:
**sparse** (matrix $A$): Substantial savings in either operations or storage can be achieved when the zero elements of $A$ are exploited during the application of Gaussian elimination to $A$, **40.2**. A large fraction of the entries of $A$ are zeros, **43**.

Ch. N p. N-3, To conform to the notation change in Chapter 4:

$A^{(k)}$     $k$th compound matrix of $A$ §1.4.2