Workshop on ATLAS OF LIE GROUPS IV
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Mathematicians are interested in measuring symmetry. This symmetry can take many forms: algebraic, geometric, or analytic. The theory of Lie groups actually exploits all three points of view. A subject that arose as part of the theory of flows stemming from differential equations, Lie groups has now developed into a very abstract form of harmonic analysis. It concerns itself with groups that are also geometric objects of a certain type. A simple example of a Lie group is the torus $S^1 \times S^1 = \{(e^{i\theta}, e^{i\psi}) : 0 \leq \theta, \psi \leq 2\pi\}$.

One method of studying the structure of a Lie group G is to study the mappings of that group into a standard, fixed object. Often we use the unitary group on n letters (called $U(n)$) as that standard object. These are the "complex rotations" in n-dimensional complex space. The Lie group theorists would like to be able, given a fixed Lie group G, to calculate all the algebraic mappings of G into $U(n)$. The space of such mappings is called the *dual* of the group. Given any Lie group, we would like to be able to calculate its dual. An effort to implement this goal is called the "Atlas of Lie Groups" project.

An important component of this project is a Web site that would provide background information about the Atlas, information about the means and methods for calculating the dual, and then a piece of software that will actually carry out the calculation of the dual for any given Lie group. The Web site is an overall archive for this subject area. It contains background papers, names and contact data for workers in the field, links to other Web sites, and tables of data about Lie groups (including root systems, classifications of representations, Cartan subgroups, Weyl groups, Kazhdan-Lusztig-Vogan polynomials, and other technical data). This should be an important tool for mathematicians of all disciplines, and for physicists and other mathematical scientists as well.

It has been known for about twenty years that, for a given Lie group, there is a finite algorithm for carrying out the dual calculation. However it is enormously difficult to describe this algorithm explicitly, and it is not clear it can be implemented by computer. There is an enormous amount of deep mathematics required to go from the existence of the algorithm to an implementation.

This is the fourth meeting at AIM of the Atlas of Lie Groups team. They now have a large Focused Research Group grant from the National Science Foundation that will enable them to carry out this program for some years to come. They have a beta version of the desired software—posted on the Web—that can perform some of the desired calculations. For example, the Kazhdan-Lusztig polynomials for a real Lie group can now be calculated.

Part of the problem here is that, typically, the dual of a given Lie group is an infinte set. It is in the nature of computer calculations that the machine can only deal with a finite amount of information, and can only present an answer that entails a finite amount of code. Part of the algorithm is the ability to describe the dual in a finite way. The problem is multi-faceted, and involves questions of abstract logic, algebra, algorithm theory, computer science, and of course Lie group theory.

Mathematicians and computer scientists from the US and Europe are participating in this important project. AIM is providing ongoing support for the effort.